37 Extension Note

July 1999

Revised Growth Intercept Models for Coastal Western Hemlock, Sitka Spruce, and Interior Spruce

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Abstract

The original growth intercept models for Sitka spruce (*Picea sitchensis* [Bong.] Carr.), coastal western hemlock (*Tsuga heterophylla* [Raf.] Sarg.), and interior spruce—white spruce (*Picea glauca* [Moench] Voss), Engelmann spruce (*P. engelmannii* Parry), and their hybrids—are dated. The original data, plus some additional data for interior spruce, were re-analyzed to incorporate new techniques into the models. The resulting models for Sitka spruce are more precise than the original models, although the revised models for western hemlock and interior spruce are slightly less precise. The advantages of the new techniques outweigh any loss of precision. The models for these species are upgraded to current standards and are consistent with models for other species.

Introduction

Growth intercept models have been developed for three tree species: Sitka spruce (*Picea sitchensis* [Bong.] Carr.) (Nigh 1996a); western hemlock (*Tsuga heterophylla* [Raf.] Sarg.) on the coast of British Columbia (Nigh 1996b); and interior spruce, which comprises white spruce (*Picea glauca* [Moench] Voss), Engelmann spruce (*P. engelmannii* Parry), and their hybrids (Nigh 1996c). These models were three of the first growth intercept models developed using variable growth intercept modelling. Since then, several advances have been made in the technique (Nigh 1997, p. 12). The growth intercept models for western hemlock, Sitka spruce, and interior spruce were revised to take advantage of some of the enhancements, and are presented in this Extension Note. These enhancements include incorporating the new model equation, extending the models to breast height age 50, and explicitly calculating the proportion of height growth between breast height ages zero and one.

Data

The stem analysis data for the western hemlock and Sitka spruce models are described by Nigh (1996a, 1996b). The interior spruce data consist of 87 plots from three sources: a height–age modelling project, a productivity study by the University of British Columbia (Wang and Klinka 1996), and a growth intercept modelling project.[1]

1 J.S. Thrower and Associates Ltd. 1995. Growth intercepts for interior spruce in north central British Columbia. B.C. Min. For., Silv. Practices Br., Victoria, B.C. Unpubl. rep.

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Methods

The stem analysis data were converted into height–breast height age data as described by Newberry (1991). The heights of the sample trees in each plot were averaged by breast height age up to breast height age 50. The average height of the trees in plot *i* at breast height age 50 is the site index of plot *i*. The growth intercepts for plot *i* were calculated as

$$GI_{i,A} = \frac{H_{i,A} - 1.3}{A - A_{i,p}} \times 100, \qquad [1]$$

where: $GI_{i,A}$ is the growth intercept (cm/yr) at breast height age A (yr); $A = 1, 2, 3, \ldots 50$; $H_{i,A}$ is the average height (m) of the sample trees at breast height age A; and $A_{i,p} = (1.3 - H_{i,0}) / (H_{i,1} - H_{i,0})$, an estimate of the proportion of the growing season between breast height ages zero and one in which the trees were less than 1.3 m.

For each breast height age between one and 50, the site index–growth intercept data were fit to

$$SI_i = 1.3 + b_0 \times GI_{i,A}^{b_1} + \varepsilon_i \qquad [2]$$

where: SI_i is the site index (m) of plot *i*; b_0 *and* b_1 are parameters; and ε_i is the random error term. The fitting was done with nonlinear least-squares regression after performing an exponential transformation on parameter b_0 to remove parameter-effects nonlinearity (see Nigh 1997, p. 7). Tests were performed to confirm the standard regression assumption of unbiased normally distributed residuals with a constant variance (Sen and Srivastava 1990; Nigh 1997).

Results

The fitted growth intercept models for coastal western hemlock, Sitka spruce, and interior spruce are presented in Tables 1, 2, and 3, respectively. These tables also show the square root of the mean squared error, which is a measure of the precision of the models. The statistical tests on the residuals (not reported) show that their mean is not significantly different from zero, they are normally distributed, and they have a common variance. Figure 1 shows the fitted growth intercept model overlaid on the data points for breast height ages 5, 15, 30, and 45 for coastal western hemlock (a), Sitka spruce (b), and interior spruce (c).

Discussion and Conclusion

The revised growth intercept models for Sitka spruce are slightly more precise than the original models, although the revised models for western hemlock and interior spruce are slightly less precise. The reduction in precision for hemlock is due to the new model formulation. This new formulation makes extrapolating the model on poor sites less uncertain, so I consider the benefit of better extrapolation to outweigh the loss in precision, which is of little practical significance. The loss in precision for interior spruce, which is also small, is an artifact of the expanded data set. The nonlinear model is more precise than a linear model when both models are fitted to the expanded interior spruce data set.

The growth intercept models for western hemlock are more precise than those for Sitka spruce except for breast height ages under six, and the interior spruce models are more precise than the hemlock models below breast height age 18 (see Figure 1 and the root mean squared errors reported in Tables 1, 2, and 3). All models appear to adequately fit the data (Figure 1). At young ages, in particular, the growth of the coastal species is more variable than that of the interior spruce. Site factors that change, such as climate, appear to significantly influence the growth of the coastal trees.

The revised models for western hemlock, Sitka spruce, and interior spruce extend the period for which the models are appropriate to breast height

age 50. Incorporating the latest growth intercept techniques into the models updates them to current standards and makes the growth intercept models consistent for all species.

Acknowledgements

Pat Martin (B.C. Ministry of Forests, Forest Practices Branch), Peter Ott (B.C. Ministry of Forests, Research Branch), and Sam Otukol (B.C. Ministry of Forests, Resources Inventory Branch) reviewed an earlier draft of this report.

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TABLE 1 *Growth intercept models for coastal western hemlock*

bh age (yr)[a]	Model	RMSE[b]	bh age (yr)	Model	RMSE
1	$\hat{SI} = 1.3 + 4.361 \times GI_1^{0.4638}$	5.614	26	$\hat{SI} = 1.3 + 0.9440 \times GI_{26}^{0.8196}$	1.735
2	$\hat{SI} = 1.3 + 3.678 \times GI_2^{0.5047}$	4.920	27	$\hat{SI} = 1.3 + 0.8852 \times GI_{27}^{0.8356}$	1.654
3	$\hat{SI} = 1.3 + 3.359 \times GI_3^{0.5302}$	4.682	28	$\hat{SI} = 1.3 + 0.8406 \times GI_{28}^{0.8485}$	1.571
4	$\hat{SI} = 1.3 + 3.221 \times GI_4^{0.5432}$	4.541	29	$\hat{SI} = 1.3 + 0.7961 \times GI_{29}^{0.8621}$	1.491
5	$\hat{SI} = 1.3 + 2.857 \times GI_5^{0.5749}$	4.188	30	$\hat{SI} = 1.3 + 0.7478 \times GI_{30}^{0.8781}$	1.385
6	$\hat{SI} = 1.3 + 2.459 \times GI_6^{0.6134}$	3.762	31	$\hat{SI} = 1.3 + 0.7026 \times GI_{31}^{0.8939}$	1.284
7	$\hat{SI} = 1.3 + 2.229 \times GI_7^{0.6373}$	3.430	32	$\hat{SI} = 1.3 + 0.6607 \times GI_{32}^{0.9095}$	1.204
8	$\hat{SI} = 1.3 + 2.149 \times GI_8^{0.6436}$	3.267	33	$\hat{SI} = 1.3 + 0.6217 \times GI_{33}^{0.9251}$	1.130
9	$\hat{SI} = 1.3 + 2.026 \times GI_9^{0.6550}$	3.143	34	$\hat{SI} = 1.3 + 0.5906 \times GI_{34}^{0.9387}$	1.057
10	$\hat{SI} = 1.3 + 1.842 \times GI_{10}^{0.6756}$	2.953	35	$\hat{SI} = 1.3 + 0.5699 \times GI_{35}^{0.9488}$	0.9717
11	$\hat{SI} = 1.3 + 1.692 \times GI_{11}^{0.6937}$	2.759	36	$\hat{SI} = 1.3 + 0.5538 \times GI_{36}^{0.9571}$	0.8842
12	$\hat{SI} = 1.3 + 1.547 \times GI_{12}^{0.7120}$	2.603	37	$\hat{SI} = 1.3 + 0.5415 \times GI_{37}^{0.9640}$	0.8067
13	$\hat{SI} = 1.3 + 1.440 \times GI_{13}^{0.7267}$	2.475	38	$\hat{SI} = 1.3 + 0.5320 \times GI_{38}^{0.9696}$	0.7360
14	$\hat{SI} = 1.3 + 1.392 \times GI_{14}^{0.7337}$	2.371	39	$\hat{SI} = 1.3 + 0.5231 \times GI_{39}^{0.9750}$	0.6691
15	$\hat{SI} = 1.3 + 1.360 \times GI_{15}^{0.7383}$	2.250	40	$\hat{SI} = 1.3 + 0.5139 \times GI_{40}^{0.9805}$	0.6036
16	$\hat{SI} = 1.3 + 1.333 \times GI_{16}^{0.7422}$	2.175	41	$\hat{SI} = 1.3 + 0.5073 \times GI_{41}^{0.9847}$	0.5414
17	$\hat{SI} = 1.3 + 1.294 \times GI_{17}^{0.7487}$	2.133	42	$\hat{SI} = 1.3 + 0.5038 \times GI_{42}^{0.9875}$	0.4812
18	$\hat{SI} = 1.3 + 1.254 \times GI_{18}^{0.7554}$	2.046	43	$\hat{SI} = 1.3 + 0.5017 \times GI_{43}^{0.9896}$	0.4207
19	$\hat{SI} = 1.3 + 1.211 \times GI_{19}^{0.7625}$	1.959	44	$\hat{SI} = 1.3 + 0.5001 \times GI_{44}^{0.9914}$	0.3671
20	$\hat{SI} = 1.3 + 1.162 \times GI_{20}^{0.7716}$	1.909	45	$\hat{SI} = 1.3 + 0.4991 \times GI_{45}^{0.9929}$	0.3154
21	$\hat{SI} = 1.3 + 1.125 \times GI_{21}^{0.7787}$	1.906	46	$\hat{SI} = 1.3 + 0.4989 \times GI_{46}^{0.9940}$	0.2613
22	$\hat{SI} = 1.3 + 1.086 \times GI_{22}^{0.7864}$	1.908	47	$\hat{SI} = 1.3 + 0.4987 \times GI_{47}^{0.9951}$	0.2043
23	$\hat{SI} = 1.3 + 1.049 \times GI_{23}^{0.7942}$	1.903	48	$\hat{SI} = 1.3 + 0.4963 \times GI_{48}^{0.9973}$	0.1483
24	$\hat{SI} = 1.3 + 1.026 \times GI_{24}^{0.7991}$	1.857	49	$\hat{SI} = 1.3 + 0.4939 \times GI_{49}^{0.9997}$	0.09467
25	$\hat{SI} = 1.3 + 0.9952 \times GI_{25}^{0.8064}$	1.793	50	$\hat{SI} = 1.3 + 0.4914 \times GI_{50}^{1.002}$	0.05734

a bh age = breast height age
b RMSE = root mean squared error

TABLE 2 *Growth intercept models for Sitka spruce*

bh age (yr)[a]	Model	RMSE[b]	bh age (yr)	Model	RMSE
1	$\hat{SI} = 1.3 + 4.367 \times GI_1^{0.5034}$	4.390	26	$\hat{SI} = 1.3 + 0.9735 \times GI_{26}^{0.8193}$	2.396
2	$\hat{SI} = 1.3 + 3.164 \times GI_2^{0.5731}$	4.156	27	$\hat{SI} = 1.3 + 0.9481 \times GI_{27}^{0.8253}$	2.300
3	$\hat{SI} = 1.3 + 3.008 \times GI_3^{0.5825}$	4.142	28	$\hat{SI} = 1.3 + 0.9226 \times GI_{28}^{0.8318}$	2.200
4	$\hat{SI} = 1.3 + 2.900 \times GI_4^{0.5904}$	4.135	29	$\hat{SI} = 1.3 + 0.8906 \times GI_{29}^{0.8403}$	2.106
5	$\hat{SI} = 1.3 + 2.810 \times GI_5^{0.5978}$	4.107	30	$\hat{SI} = 1.3 + 0.8782 \times GI_{30}^{0.8435}$	2.029
6	$\hat{SI} = 1.3 + 2.771 \times GI_6^{0.6009}$	4.100	31	$\hat{SI} = 1.3 + 0.8574 \times GI_{31}^{0.8493}$	1.952
7	$\hat{SI} = 1.3 + 2.889 \times GI_7^{0.5891}$	4.124	32	$\hat{SI} = 1.3 + 0.8196 \times GI_{32}^{0.8604}$	1.862
8	$\hat{SI} = 1.3 + 3.042 \times GI_8^{0.5737}$	4.180	33	$\hat{SI} = 1.3 + 0.7985 \times GI_{33}^{0.8670}$	1.769
9	$\hat{SI} = 1.3 + 3.095 \times GI_9^{0.5696}$	4.223	34	$\hat{SI} = 1.3 + 0.7799 \times GI_{34}^{0.8731}$	1.672
10	$\hat{SI} = 1.3 + 3.049 \times GI_{10}^{0.5674}$	4.230	35	$\hat{SI} = 1.3 + 0.7638 \times GI_{35}^{0.8786}$	1.572
11	$\hat{SI} = 1.3 + 2.931 \times GI_{11}^{0.5745}$	4.211	36	$\hat{SI} = 1.3 + 0.7491 \times GI_{36}^{0.8839}$	1.470
12	$\hat{SI} = 1.3 + 2.780 \times GI_{12}^{0.5849}$	4.173	37	$\hat{SI} = 1.3 + 0.7349 \times GI_{37}^{0.8891}$	1.375
13	$\hat{SI} = 1.3 + 2.561 \times GI_{13}^{0.6025}$	4.063	38	$\hat{SI} = 1.3 + 0.7227 \times GI_{38}^{0.8938}$	1.285
14	$\hat{SI} = 1.3 + 2.312 \times GI_{14}^{0.6250}$	3.908	39	$\hat{SI} = 1.3 + 0.7112 \times GI_{39}^{0.8985}$	1.201
15	$\hat{SI} = 1.3 + 2.100 \times GI_{15}^{0.6462}$	3.782	40	$\hat{SI} = 1.3 + 0.6967 \times GI_{40}^{0.9045}$	1.113
16	$\hat{SI} = 1.3 + 1.899 \times GI_{16}^{0.6687}$	3.667	41	$\hat{SI} = 1.3 + 0.6770 \times GI_{41}^{0.9125}$	1.014
17	$\hat{SI} = 1.3 + 1.730 \times GI_{17}^{0.6897}$	3.547	42	$\hat{SI} = 1.3 + 0.6551 \times GI_{42}^{0.9217}$	0.9135
18	$\hat{SI} = 1.3 + 1.603 \times GI_{18}^{0.7068}$	3.436	43	$\hat{SI} = 1.3 + 0.6351 \times GI_{43}^{0.9305}$	0.8221
19	$\hat{SI} = 1.3 + 1.500 \times GI_{19}^{0.7215}$	3.314	44	$\hat{SI} = 1.3 + 0.6148 \times GI_{44}^{0.9396}$	0.7306
20	$\hat{SI} = 1.3 + 1.400 \times GI_{20}^{0.7369}$	3.180	45	$\hat{SI} = 1.3 + 0.5924 \times GI_{45}^{0.9498}$	0.6211
21	$\hat{SI} = 1.3 + 1.303 \times GI_{21}^{0.7531}$	3.038	46	$\hat{SI} = 1.3 + 0.5698 \times GI_{46}^{0.9605}$	0.4991
22	$\hat{SI} = 1.3 + 1.206 \times GI_{22}^{0.7705}$	2.876	47	$\hat{SI} = 1.3 + 0.5489 \times GI_{47}^{0.9710}$	0.3691
23	$\hat{SI} = 1.3 + 1.119 \times GI_{23}^{0.7875}$	2.722	48	$\hat{SI} = 1.3 + 0.5301 \times GI_{48}^{0.9808}$	0.2394
24	$\hat{SI} = 1.3 + 1.050 \times GI_{24}^{0.8021}$	2.590	49	$\hat{SI} = 1.3 + 0.5143 \times GI_{49}^{0.9897}$	0.1212
25	$\hat{SI} = 1.3 + 1.004 \times GI_{25}^{0.8123}$	2.488	50	$\hat{SI} = 1.3 + 0.4986 \times GI_{50}^{0.9987}$	0.04830

a bh age = breast height age
b RMSE = root mean squared error

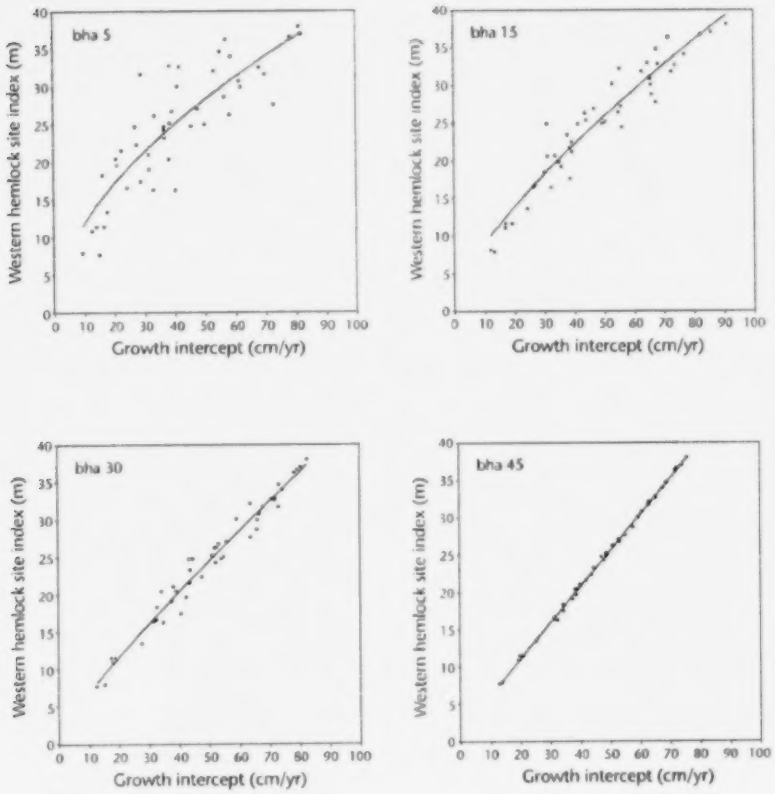
TABLE 3 *Growth intercept models for interior spruce*

bh age (yr)[a]	Model	RMSE[b]	bh age (yr)	Model	RMSE
1	$\hat{SI} = 1.3 + 4.050 \times GI_1^{0.4630}$	2.767	26	$\hat{SI} = 1.3 + 1.139 \times GI_{26}^{0.7727}$	1.580
2	$\hat{SI} = 1.3 + 3.215 \times GI_2^{0.5222}$	2.517	27	$\hat{SI} = 1.3 + 1.069 \times GI_{27}^{0.7891}$	1.515
3	$\hat{SI} = 1.3 + 2.917 \times GI_3^{0.5509}$	2.442	28	$\hat{SI} = 1.3 + 1.008 \times GI_{28}^{0.8045}$	1.447
4	$\hat{SI} = 1.3 + 2.768 \times GI_4^{0.5674}$	2.397	29	$\hat{SI} = 1.3 + 0.9554 \times GI_{29}^{0.8188}$	1.382
5	$\hat{SI} = 1.3 + 2.749 \times GI_5^{0.5683}$	2.400	30	$\hat{SI} = 1.3 + 0.9067 \times GI_{30}^{0.8327}$	1.325
6	$\hat{SI} = 1.3 + 2.724 \times GI_6^{0.5671}$	2.384	31	$\hat{SI} = 1.3 + 0.8666 \times GI_{31}^{0.8448}$	1.273
7	$\hat{SI} = 1.3 + 2.685 \times GI_7^{0.5672}$	2.367	32	$\hat{SI} = 1.3 + 0.8366 \times GI_{32}^{0.8541}$	1.222
8	$\hat{SI} = 1.3 + 2.646 \times GI_8^{0.5675}$	2.345	33	$\hat{SI} = 1.3 + 0.8074 \times GI_{33}^{0.8636}$	1.165
9	$\hat{SI} = 1.3 + 2.572 \times GI_9^{0.5724}$	2.313	34	$\hat{SI} = 1.3 + 0.7745 \times GI_{34}^{0.8749}$	1.108
10	$\hat{SI} = 1.3 + 2.503 \times GI_{10}^{0.5772}$	2.276	35	$\hat{SI} = 1.3 + 0.7386 \times GI_{35}^{0.8877}$	1.038
11	$\hat{SI} = 1.3 + 2.469 \times GI_{11}^{0.5780}$	2.261	36	$\hat{SI} = 1.3 + 0.7095 \times GI_{36}^{0.8985}$	0.9617
12	$\hat{SI} = 1.3 + 2.419 \times GI_{12}^{0.5812}$	2.228	37	$\hat{SI} = 1.3 + 0.6861 \times GI_{37}^{0.9075}$	0.8894
13	$\hat{SI} = 1.3 + 2.334 \times GI_{13}^{0.5892}$	2.188	38	$\hat{SI} = 1.3 + 0.6651 \times GI_{38}^{0.9159}$	0.8316
14	$\hat{SI} = 1.3 + 2.259 \times GI_{14}^{0.5963}$	2.174	39	$\hat{SI} = 1.3 + 0.6409 \times GI_{39}^{0.9261}$	0.7735
15	$\hat{SI} = 1.3 + 2.178 \times GI_{15}^{0.6044}$	2.165	40	$\hat{SI} = 1.3 + 0.6157 \times GI_{40}^{0.9372}$	0.7032
16	$\hat{SI} = 1.3 + 2.079 \times GI_{16}^{0.6153}$	2.133	41	$\hat{SI} = 1.3 + 0.5945 \times GI_{41}^{0.9469}$	0.6364
17	$\hat{SI} = 1.3 + 1.965 \times GI_{17}^{0.6294}$	2.086	42	$\hat{SI} = 1.3 + 0.5721 \times GI_{42}^{0.9575}$	0.5698
18	$\hat{SI} = 1.3 + 1.857 \times GI_{18}^{0.6442}$	2.041	43	$\hat{SI} = 1.3 + 0.5508 \times GI_{43}^{0.9681}$	0.5028
19	$\hat{SI} = 1.3 + 1.772 \times GI_{19}^{0.6562}$	2.002	44	$\hat{SI} = 1.3 + 0.5347 \times GI_{44}^{0.9765}$	0.4358
20	$\hat{SI} = 1.3 + 1.673 \times GI_{20}^{0.6711}$	1.954	45	$\hat{SI} = 1.3 + 0.5208 \times GI_{45}^{0.9841}$	0.3619
21	$\hat{SI} = 1.3 + 1.578 \times GI_{21}^{0.6881}$	1.901	46	$\hat{SI} = 1.3 + 0.5101 \times GI_{46}^{0.9902}$	0.2873
22	$\hat{SI} = 1.3 + 1.486 \times GI_{22}^{0.7016}$	1.838	47	$\hat{SI} = 1.3 + 0.5047 \times GI_{47}^{0.9935}$	0.2193
23	$\hat{SI} = 1.3 + 1.394 \times GI_{23}^{0.7186}$	1.777	48	$\hat{SI} = 1.3 + 0.5006 \times GI_{48}^{0.9963}$	0.1530
24	$\hat{SI} = 1.3 + 1.301 \times GI_{24}^{0.7371}$	1.710	49	$\hat{SI} = 1.3 + 0.4970 \times GI_{49}^{0.9989}$	0.08392
25	$\hat{SI} = 1.3 + 1.215 \times GI_{25}^{0.7551}$	1.644	50	$\hat{SI} = 1.3 + 0.4934 \times GI_{50}^{1.000}$	0.02886

a bh age = breast height age
b RMSE = root mean squared error

FIGURE 1 *Growth intercept curves overlaid on the data points for breast height ages 5, 15, 30, and 45. Part (a) is western hemlock, part (b) is Sitka spruce, and part (c) is interior spruce.*

part (a)



part (b)

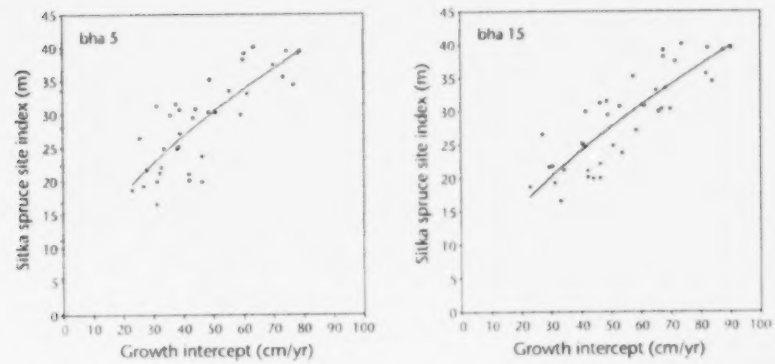
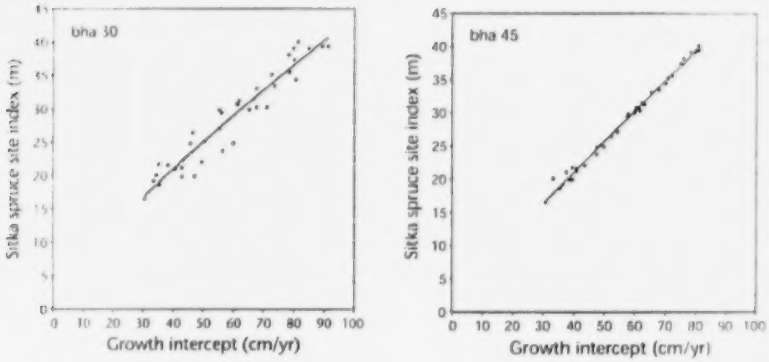


FIGURE 1 *Continued*



part (c)